Abstract: The stochastic partial differential equation (SPDE) approach is widely used for modeling large spatial datasets. It is based on representing a Gaussian random field u on $\mathbb{R}^d$ as the solution of an elliptic SPDE $L^\beta u = \mathcal{W}$ where L is a second-order differential operator, $2\beta \in \mathbbm{N}$ is a positive parameter that controls the smoothness of u and $\mathcal{W}$ is Gaussian white noise. A few approaches have been suggested in the literature to extend the approach to allow for any smoothness parameter satisfying $\beta>d/4$. Even though those approaches work well for simulating SPDEs with general smoothness, they are less suitable for Bayesian inference since they do not provide approximations which are Gaussian Markov random fields (GMRFs) as in the original SPDE approach. We address this issue by proposing a new method based on approximating the covariance operator $L^{-2\beta}$ of the Gaussian field u by a finite element method combined with a rational approximation of the fractional power. This results in a numerically stable GMRF approximation which can be combined with the integrated nested Laplace approximation (INLA) method for fast Bayesian inference. A rigorous convergence analysis of the method is performed and the accuracy of the method is investigated with numerical experiments. Finally, we illustrate the approach and corresponding implementation in the R package rSPDE via an application to precipitation data which is analyzed by combining the rSPDE package with the R-INLA software for full Bayesian inference.

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